The AC Resistance of a Microstripline and Its Ground Plane

by R. Faraji-Dana and Y.L. Chow
Electrical Engineering Dept.
University of Waterloo
Waterloo, Ont., Canada N2L 3G1

Abstract

It is found that the RF resistance of a microstripline is substantially higher than the theoretical value from the skin effect of the strip alone. Based on rigorous analyses, this paper finds that the extra resistance is caused mainly by (i) The concentration of grounded plane current under a wide microstrip, and (ii) the concentration of the strip current towards the ground.

Introduction

It was found recently by Verner et al [1] that the experimentally measured resistance of a microstripline is about 40 to 80% above its theoretically calculated value based on the strip skin effect alone. A similar phenomenon was also found by Stubbs et al [2] in the experimental and theoretical values of resistances in a microwave integrated amplifier. It is important to understand this apparent rise in AC resistances for better modeling in the MIC-CAD softwares.

We find that the extra AC resistance comes from three main sources: (i) the familiar Meixner condition of higher magnetic field at the corners of a rectangular strip, but more importantly, (ii) the shifting of current to the bottom of the strip due to the presence of the ground plane and (iii) the extra resistive loss due to the concentration of ground plane current under the microstripline. The last two losses especially dominate the rise in AC resistance for a wide microstripline, i.e. a line with its width larger than the substrate height. A 50 Ω microstripline with Duroid substrate (ε_r = 2.3) is in fact a wide line.

The above conclusions are quantitative and accurate as much of the derivations are done rigorously, using techniques of variable separation and spectral space analyses for lossy ground [3].

Parts of the derivations were presented [4][5]. In their totality, however, they are lengthy and are better published in journals. For this Conference, therefore, it is better to give just the basic formulation of the derivations, and then discuss the results arrived and compare with the available results in the literature. Some important literature are works by Pucel [6] and Djordjevic et al [7].

The derivation and discussions can be divided into three following stages:

(i) The AC resistance of an isolated strip

The isolated strip of thickness t and width w is shown in Fig. 1. The interior of the strip is governed by the equations

$$\nabla^2 E_z + T^2 E_z = 0 \qquad (1)$$

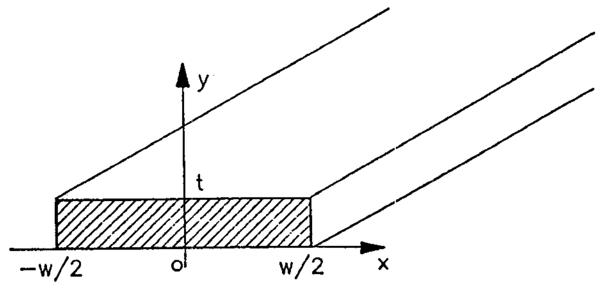

Fig. 1 An isolated strip.

$$\text{where} \quad T^2 = -j\omega\mu\sigma, \qquad (1a)$$

$$J_z = \sigma E_z. \qquad (2)$$

The exterior is governed by

$$E_z = \int\int_{strip} g J_z d\vec{s}' \qquad (3)$$

$$\text{where } g = \frac{j\omega\mu}{2\pi}\ln r \quad , \quad \text{for} \quad w \ll \lambda. \qquad (3a)$$

With rectangular strip, the general solution of Eq. (1) is obtained through variable separation [4]. This general solution plus Eq. (2) and (3), are then solved numerically, subject to: (a) the condition of continuous E_z across the strip boundary, and (b) an external E_z^{ext} excitation forcing the current J_z along the strip.

The resulted current distribution satisfies the Meixner's edge condition. Fig. 2 plots the AC resistance (including Meixner's condition) versus the "normalized frequency" P. P, a frequently used parameter, is given by

$$P = \left(\sqrt{\frac{2}{\pi}}\right)\left(\frac{\sqrt{A}}{\delta}\right) = \sqrt{2\mu\sigma f A} \qquad (4).$$

The agreement with the experimental results of Haefner [8] is observed to be excellent.

(ii) The AC resistance of the strip of the Microstripline

The microstripline is shown in Fig. 3. The interiors of the strip is governed by the same equations (1) to (3) except that Eq. (1a) becomes

$$T^2 = -j\omega\mu\sigma - \beta^2 \doteqdot -j\omega\mu\sigma, \quad \text{for large} \quad \sigma, \qquad (5)$$

where β is the propagation constant of the microstripline.

The resistive loss is caused by current, which is affected mainly by the magnetic field, and which is, in turn, **not** much affected by the dielectric substrate. Hence we may assume

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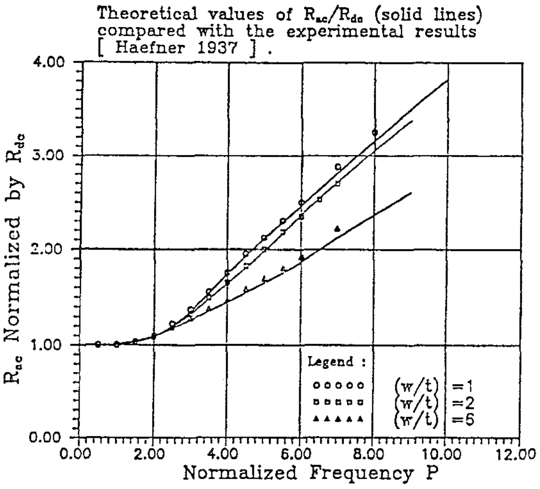

Fig. 2 The AC resistance of an isolated strip.

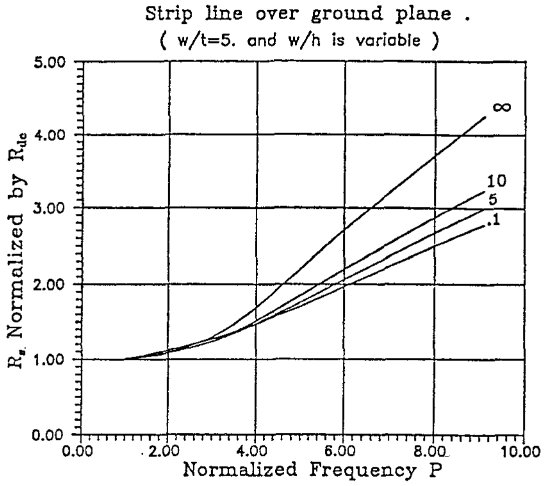

Fig. 4 The AC resistance of the strip of a microstripline.

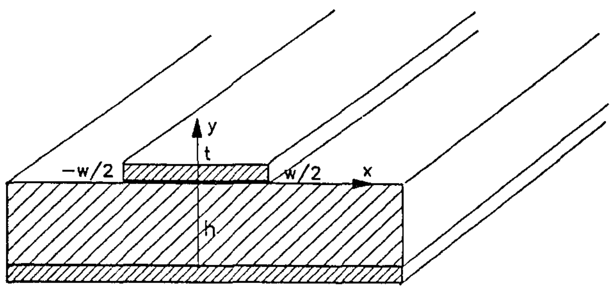

Fig. 3 A microstripline.

the substrate dielectric constant be unity to get, in place of (3a), the Green's function

$$g = \frac{j\omega\mu}{2\pi}\left[\ln\left(\frac{r'}{r}\right) + \int_0^\infty f_1(\lambda) e^{-u_0 y}\cos\lambda(x-x')\,d\lambda\right], \qquad (6)$$

i.e. the Sommerfeld Green's function with lossy ground [3].

With the same boundary condition and excitation the solution to Eq. (1) to (3) for the microstripline is obtained. Similar to Fig. 2, Fig. 4 plots the normalized AC resistance versus the normalized frequency P for different strip height h above the ground plane. For large width w (i.e. $w/h \to \infty$ in Fig. 4), most current concentrates in the bottom of the wide strip (cf. Fig. 5). As expected therefore, Fig. 4 shows a larger ratio R_s/R_{dc} for wide strip.

For extremely wide strip all current concentrates at the bottom of the strip. Therefore the AC resistance can be twice as large as AC resistance for the same strip without the ground plane.

(iii) The AC resistance of the ground plane of the microstripline

As shown in Fig. 5, for wide strip, the current not only concentrates at the bottom of the strip, the return current in the ground plane also concentrates directly under. Hence the AC resistance of the ground plane can be almost as large as that of the strip itself [5].

The AC resistance of the ground plane is calculated from the surface impedance and the surface magnetic field on the ground plane. The surface magnetic field is obtained from the known current in the strip from (ii) and the Sommerfeld integral similar to Eq. (3b).

The result is illustrated in Fig. 6, with the ratio of the AC resistances of the ground plane and the strip, versus the strip width. As indicated in the first paragraph above. This ratio approaches unity for wide strips.

Discussions

This paper points out that rigorous analytical results can be obtained for the total AC resistance R_t of a microstripline, since R_t is a sum of R_g of the ground plane and R_s of the strip.

The rigorous results indicate that for wide strips, R_g approaches R_s, and R_s can be twice as as R_{ac}, the ac resistance of the same strip isolated (i.e. without ground plate). This means that the AC resistance of the microstrip R_t can be four times as large as the AC resistance R_{ac} of the same strip isolated. In turn, R_{ac} is larger than R_{dc}.

Finally it is to point out that this paper is on straight microstripline. However, it is expected similar conclusions as the last paragraph can be reached for curved and bent microstriplines in a circuit.

References

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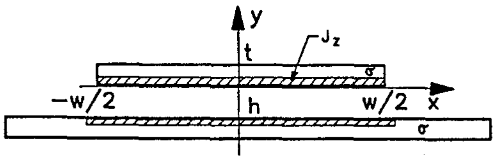

Fig. 5 A wide microstripline.

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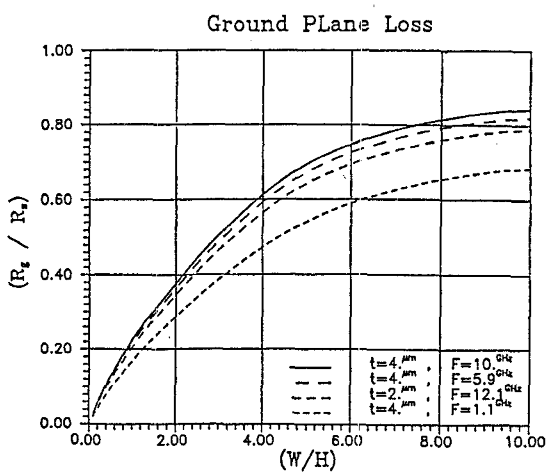

Fig. 6 The AC resistance ratio between the ground plane and the strip.